

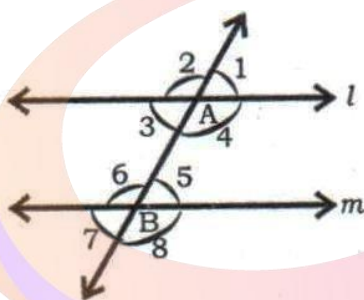
NOTES

- NOTES WITH MIND MAPS -
MATHEMATICS
(LINES AND ANGLES)



Lines and Angles

- Two angles are said to be complementary if the sum of their measures is 90° .
- Two angles are said to be supplementary if the sum of their measures is 180° .
- Two angles with the same vertex, one arm common and the other arms lying on opposite sides of the common arm are called adjacent angles.
- Two adjacent angles are said to form a linear pair of angles if their non-common arms are two opposite rays.
- The sum of the measures of a linear pair angles is always 180° .
- The sum of all angles formed on the same side of a line at a given point on the line is 180° .
- The sum of all angles around a point is 360° .
- Two angles are called a pair of vertically opposite angles if their arms from two pairs of opposite rays.
- If two lines intersect each other then the vertically opposite angles are equal.
- Two lines 'l' and 'm' are said to be parallel, if they lie in the same plane and do not intersect when produced however far on either side and is written as $l \parallel m$.
- A line which intersects two or more lines at distinct points is called a transversal.
- Angles made by parallel lines and transversal:

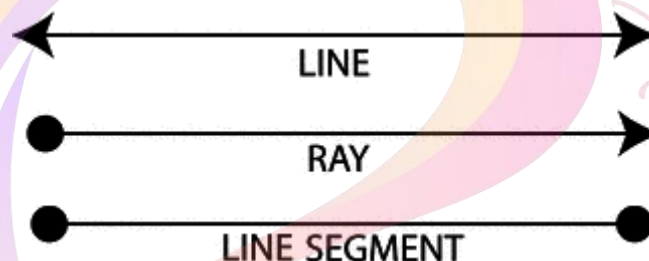


- $\angle 1, \angle 5$, $\angle 2, \angle 6$, $\angle 3, \angle 7$ and $\angle 4, \angle 8$ are called pair of corresponding angles. Check for angle sign
 - $\angle 3, \angle 5$ and $\angle 4, \angle 6$ are called pair of alternate interior angles or simply alternate angles.
 - $\angle 4, \angle 5$ and $\angle 3, \angle 6$ are called pairs of interior angles on the same side of the transversal.
- If two parallel lines are cut by a transversal, then
 - The alternate angles are equal
 - The corresponding angles are equal and
 - The interior angles on the same side of the transversal are supplementary.

14. Two lines are parallel if, when they are cut by a transversal, they make a pair of
 - i. Corresponding angles equal or
 - ii. Alternate angles equal or
 - iii. Interior angles on the same side of the transversal supplementary.
15. Two lines which are parallel to the same line are parallel to each other.
16. If three parallel lines are intersected by two transversals, the ratio of the intercepts made on one transversal is the same as the ratio of the intercepts made on the other transversal.

Line, line segment and ray

- If we take a point and draw a straight path that extends endlessly on both the sides, then the straight path is called as a line.
- A ray is a part of a line with one endpoint.
- A line segment is a part of a line with two endpoints.

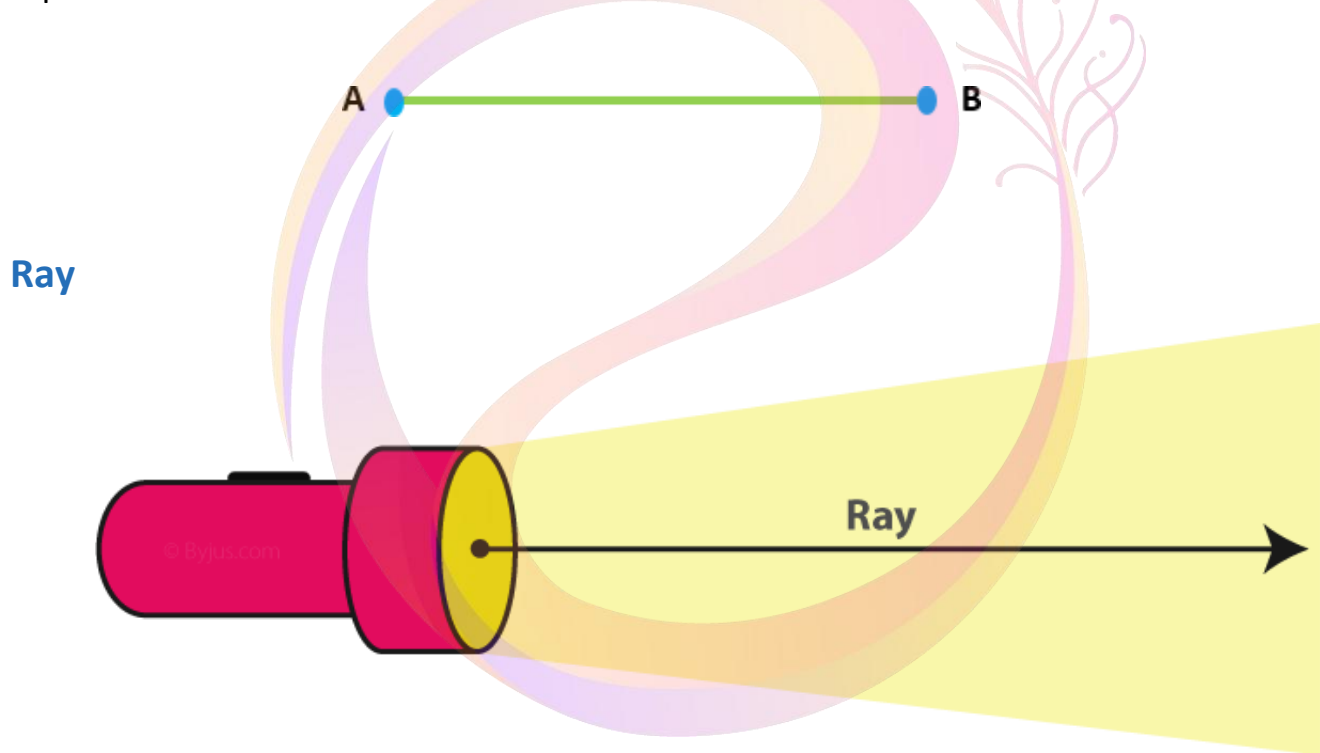


We come across many shapes in daily life. Consider a triangle, a square or any other shape. To draw any of these figures, one begins with a line or a line segment or a curve. Depending upon the number and arrangement of these lines we get distinct types of shapes and figures. For example, a triangle is a figure enclosed by 3 line segments, Pentagon is a polygon bounded by 5 line segments and so on. We have already discussed line now let us see what is line segment and ray.

Line Segment



A line segment is a part of a line having two endpoints. Figures such as a triangle, polygon, hexagon, square are made of different numbers of line segments. The measure of a line segment is called its length. In contrast to the infinitely extending line, a line segment has a fixed length and can be measured easily. A line segment with A and B as two endpoints is represented as \overline{AB}



Ray is another part of a line. It is a combination of a line and a line segment that has an infinitely extending end and one terminating end. As its one end is non-terminating, its length cannot be measured. A ray is represented by

\overrightarrow{AB}

where one end is symbolized by an endpoint and the infinitely extending part by an arrow.

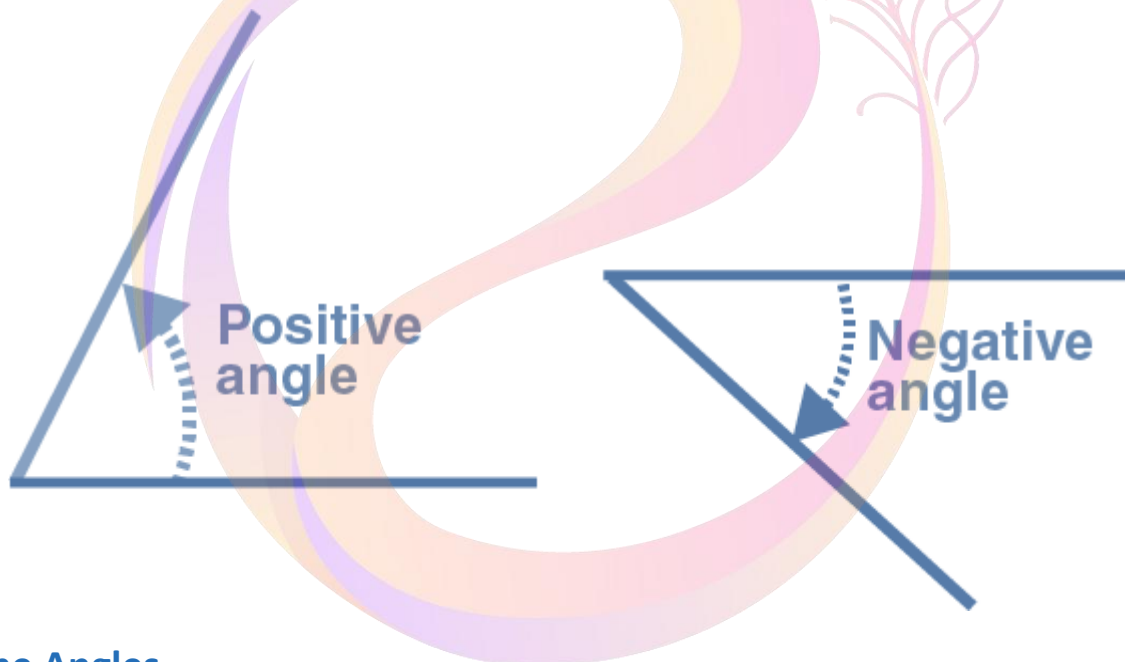


Angles

What is an angle? In Plane Geometry, a figure which is formed by two rays or lines that shares a common endpoint is called an angle. The word “angle” is derived from the Latin word “angulus”, which means “corner”. The two rays are called the sides of an angle, and the common endpoint is called the vertex. The angle that lies in the plane does not have to be in the Euclidean space. In case if the angles are formed by the intersection of two planes in the Euclidean or the other space, the angles are considered dihedral angles. The angle is represented using the symbol “ \angle ”. The angle measurement between the two rays can be denoted using the Greek letter θ , α , β etc. If the angles are measured from a line, we can find two different types of angles, such as a positive angle and a negative angle.

Positive Angle: If the angle goes in counterclockwise, then it is called a positive angle.

Negative Angle: If the angle goes clockwise direction, then it is called a negative angle.



Label the Angles

There are two different ways to label the angles. They are:

Method 1: Give a name to the angle. Generally, the angle is named using the lower case letter like “a”, “x”, etc or by using the Greek Letters alpha (α), beta (β), theta (θ), etc.

Method 2: By using the three letters on the shapes, we can define the angle. The middle letter should be the vertex (actual angle).

For example, ABC is a triangle. To represent the angle A is equal to 60 degrees, we can define it as $\angle BAC = 60^\circ$

Measure the Angle

The angles are generally measured in degrees ($^{\circ}$). An important geometrical tool that helps to measure the angles in degree is a “protractor”. A protractor has two sets of numbers going in opposite directions. One set goes from 0 to 180 degree on the outer rim and the other set goes from 180 to 0 degree on the inner rim.

Types of Angles

The angles are classified under the following types:

Acute Angle – an angle measure less than 90 degrees

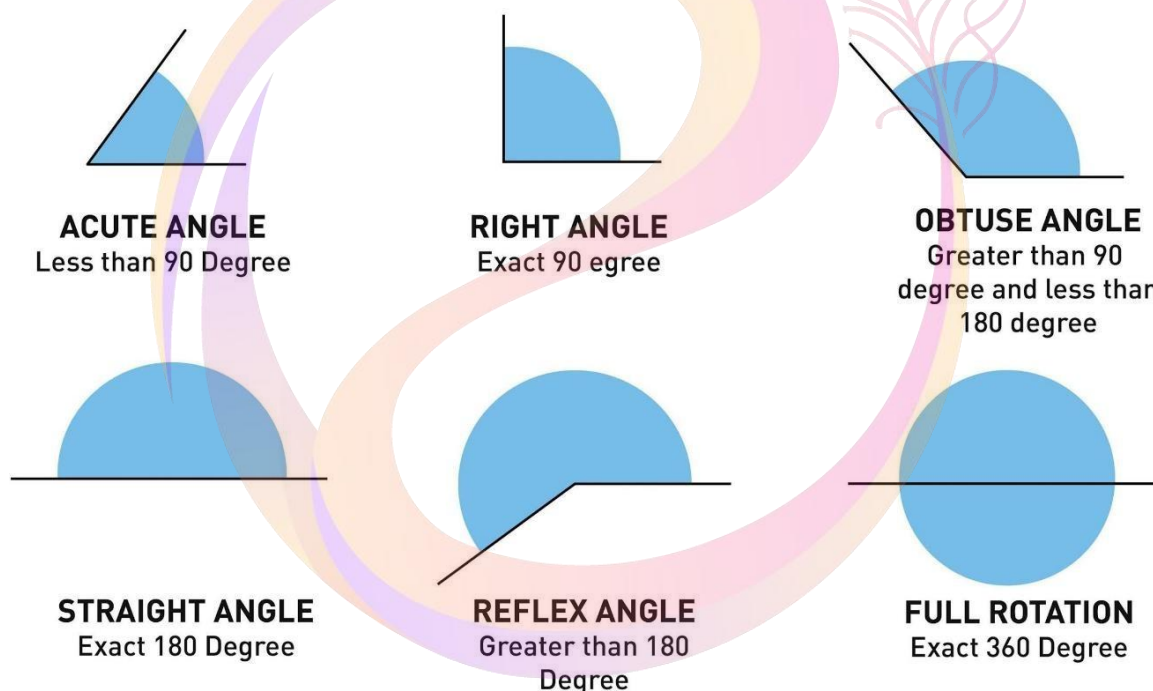
Right Angle – an angle is exactly at 90 degrees

Obtuse Angle – an angle whose measure is greater than 90 degrees and less than 180 degrees

Straight Angle – an angle which is exactly at 180 degrees

Reflex Angle – an angle whose measure is greater than 180 degrees and less than 360 degrees

Full Angle – an angle whose measure is exactly at 360 degrees



Based on these angles and the lines, it is further classified into different types such as complementary angles, supplementary angles, adjacent angles, vertical angles, alternate interior angles, alternate exterior angles, and so on.

- Corresponding Angles
- Alternate Interior Angles
- Alternate Exterior Angles
- Interior Angles on the Same Side of Transversal

- Supplementary Angles
- Adjacent Angles
- Vertical Angles

Now let us discuss some of the important theorems based on the lines and angles:

- If two parallel lines are cut by a transversal, then the alternate interior angles are of the same measure.
- If two parallel lines are cut by a transversal, then the alternate exterior angles are of the same measure.
- If two parallel lines are cut by a transversal, then the corresponding angles are of the same measure.
- If two parallel lines are cut by a transversal, then the interior angles on the same side of the transversal are supplementary.
- Vertical angles are congruent when the straight line intersects the lines. The lines may be either parallel or non-parallel

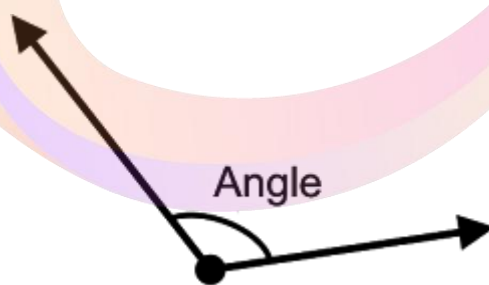
Properties of Angles

The following are the important properties of angles:

The sum of all the angles on one side of a straight line is always equal to 180 degrees,

The sum of all the angles around the point is always equal to 360 degrees.

- An angle is formed when two rays originate from the same end point.
- The rays making an angle are called the arms of the angle.
- The end point is called the vertex of the angle.

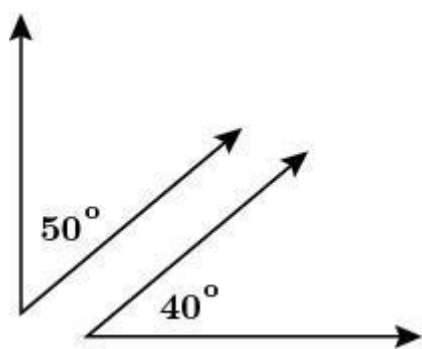


Complementary Angles

Two angles whose sum is 90° are called complementary angles.

Example: $50^\circ + 40^\circ = 90^\circ$

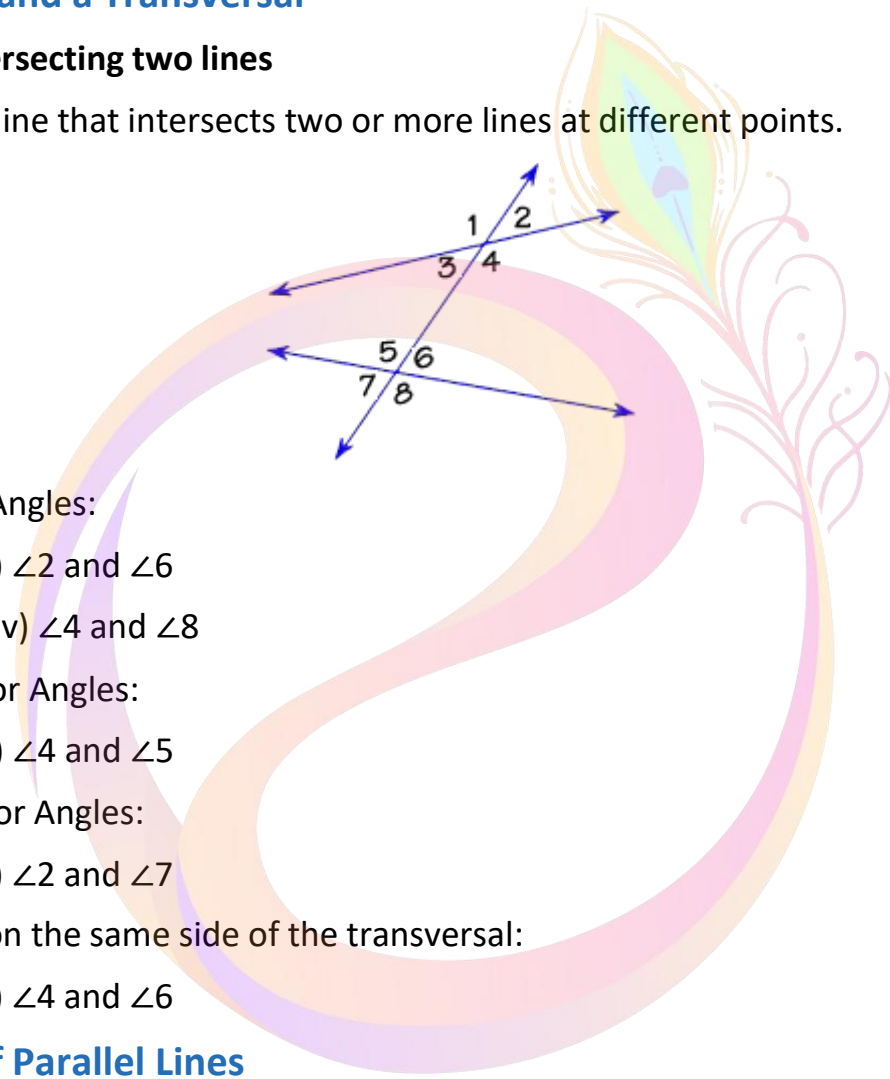
$\therefore 50^\circ$ and 40° angles are complementary angles.



Parallel Lines and a Transversal

Transversal intersecting two lines

Transversal is a line that intersects two or more lines at different points.



Corresponding Angles:

- (i) $\angle 1$ and $\angle 5$ (ii) $\angle 2$ and $\angle 6$
- (iii) $\angle 3$ and $\angle 7$ (iv) $\angle 4$ and $\angle 8$

Alternate Interior Angles:

- (i) $\angle 3$ and $\angle 6$ (ii) $\angle 4$ and $\angle 5$

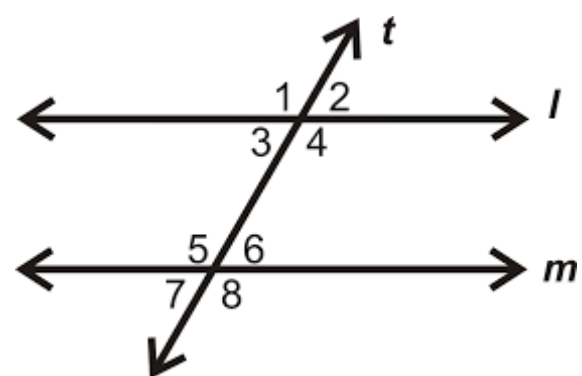
Alternate Exterior Angles:

- (i) $\angle 1$ and $\angle 8$ (ii) $\angle 2$ and $\angle 7$

Interior angles on the same side of the transversal:

- (i) $\angle 3$ and $\angle 5$ (ii) $\angle 4$ and $\angle 6$

Transversal of Parallel Lines



If a transversal intersects two parallel lines, then each pair of corresponding angles is equal.

- (i) $\angle 1 = \angle 5$ (ii) $\angle 2 = \angle 6$
 (iii) $\angle 3 = \angle 7$ (iv) $\angle 4 = \angle 8$

If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.

- (i) $\angle 3 = \angle 6$ (ii) $\angle 4 = \angle 5$

If a transversal intersects two parallel lines, then each pair of interior angles on the same side of the transversal is supplementary.

- (i) $\angle 3 + \angle 5 = 180^\circ$ (ii) $\angle 4 + \angle 6 = 180^\circ$

Checking if two or more lines are parallel

There are three conditions to check whether the two lines are parallel. They are:

- (i) If a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines are parallel to each other.
 (ii) If a transversal intersects two lines such that a pair of alternate interior angles is equal, then the two lines are parallel.
 (iii) If a transversal intersects two lines such that a pair of interior angles on the same side of the transversal is supplementary, then the two lines are parallel.

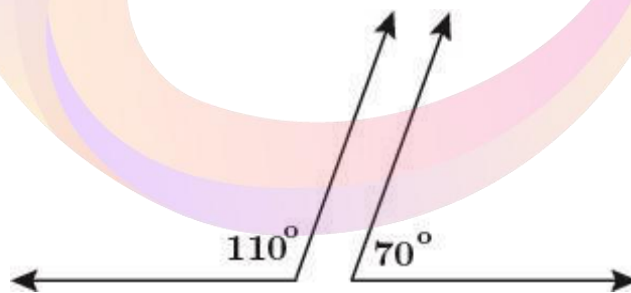
Intersecting Lines and Pairs of Angles

Supplementary angles

Two angles whose sum is 180° are called supplementary angles.

Example: $110^\circ + 70^\circ = 180^\circ$

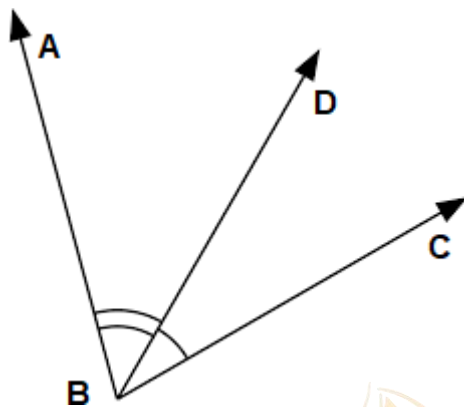
$\therefore 110^\circ$ and 70° angles are supplementary angles.



Adjacent Angles

Two angles are adjacent, if they have

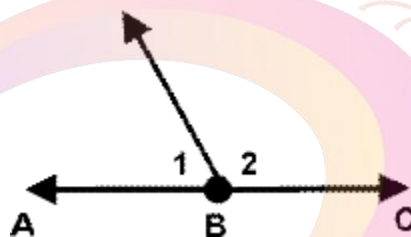
- (i) A common vertex
 (ii) A common arm
 (iii) Their non-common arms on different sides of the common arm.



Here $\angle ABD$ and $\angle DBC$ are adjacent angles.

Linear Pair

Linear pair of angles are adjacent angles whose sum is equal to 180° .

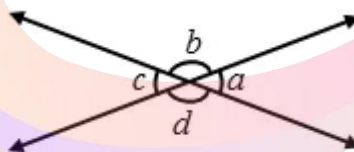


Here, 1 and 2 are linear pair of angles.

Vertically Opposite Angles

Vertically opposite angles are formed when two straight lines intersect each other at a common point.

Vertically opposite angles are equal.



Here, the following pairs of angles are vertically opposite angles.

- (i) a and c
- (ii) b and d